Functions of a Complex Variable
Complex Integrals 2

Contour integrals

Exercise 1

1. Evaluate the contour integral

$$\int_{\mathcal{C}} \bar{z}\, dz$$

where, $\mathcal{C}$ is the lines $A \to B \to C \to A$ / $A = 0$; $B = 1$; $C = i$.

2. Evaluate the contour integral $\int_{\mathcal{C}} \frac{z^2-1}{z} dz$ where, the contour $\mathcal{C}$ is:
   a. the semicircle $z = 2e^{i\theta}$ with $0 \leq \theta \leq \pi$;
   b. the semicircle $z = 2e^{i\theta}$ with $\pi \leq \theta \leq 2\pi$;
   c. the circle $z = 2e^{i\theta}$ with $0 \leq \theta \leq 2\pi$.

Cauchy Integral Theorem & Cauchy Integral Formula

Exercise 2

1. Evaluate

$$\int_{\mathcal{C}} \frac{2\, dz}{z^2 - 1}$$

where, $\mathcal{C}$ is the positively oriented circle of radius $\frac{1}{2}$, center 1.

2. Evaluate

$$\int_{\mathcal{C}} e^z \div \frac{1}{z} dz = 0$$

where, C is the lower half of the circle with radius 1, center 0, negatively oriented.