

## Functions of a Complex Variable Complex Integrals 1

Contour integrals

### Exercise 1

Evaluate the integral

$$\int_{\mathcal{C}} \operatorname{Re}(z) dz$$

for the following contours  $\mathcal{C}$  from  $-4$  to  $4$ :

- The line segments from  $-4$  to  $-4 - 4i$  to  $4 - 4i$  to  $4$ ;
- the lower half of the circle with radius  $4$ , center  $0$ ;
- the upper half of the circle with radius  $4$ , center  $0$ .
- What conclusions (if any) can you draw about the function  $f(z) = \operatorname{Re}(z)$  from this?

Cauchy Integral Formula

### Exercise 2

Evaluate

$$\int_{\mathcal{C}} \frac{\sin z}{(z + 1)^7} dz$$

where,  $\mathcal{C}$  is the circle of radius  $5$ , center  $0$ , positively oriented.

Cauchy Integral Theorem

### Exercise 3

Apply Cauchy Integral Theorem to show that

$$\int_{\mathcal{C}} \frac{z^3}{z^2 + 5z + 6} dz = 0$$

when  $\mathcal{C}$  is the unit circle  $|z| = 1$ , in either direction.