Complex Numbers – Polar Form

Exercise

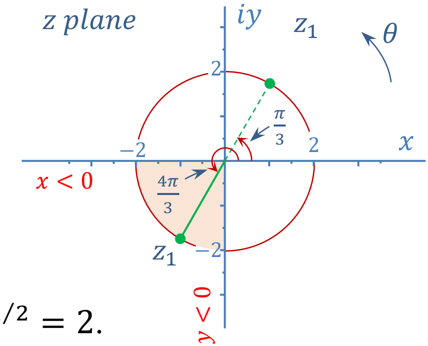
Transform the following complex numbers to polar form

$$z = r(\cos\theta + i\sin\theta)$$

with $\theta \in [0,2\pi]$.

1. $z_1 = -1 - \sqrt{3}i$.
2. $z_2 = 2 - i$.
3. $z_3 = -1 - 2i$.
4. $z_4 = -\sqrt{6} + \sqrt{2}i$.

Complex Numbers – Polar Form 1



1. $z_1 = -1 - \sqrt{3}i$

$z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ with

$$r_1 = |z_1| = (z_1\bar{z_1})^{1/2} = (x_1^2 + y_1^2)^{1/2},$$

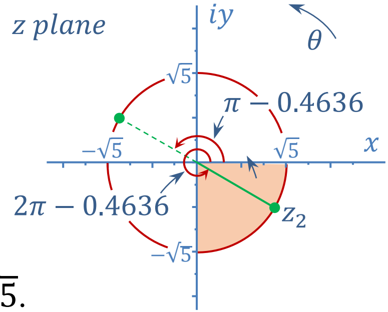
$$r_1 = \left[(-1)^2 + \left(-\sqrt{3}\right)^2\right]^{1/2} = [1 + 3]^{1/2} = 2.$$

$$\tan\theta_1 = \frac{y_1}{x_1} = \frac{-\sqrt{3}}{-1} = \sqrt{3} > 0 \Longrightarrow \begin{cases} \theta_1 \in \left[0, \frac{\pi}{2}\right] \\ \theta_1 \in \left[\pi, \frac{3\pi}{2}\right] \end{cases} \Longrightarrow \begin{cases} \theta_1 = \frac{\pi}{3} \\ \theta_1 = \frac{\pi}{3} + \pi \end{cases}.$$

$$\begin{cases} y_1 < 0 \\ x_1 < 0 \end{cases} \Longrightarrow \theta_1 \in \left[\pi, \frac{3\pi}{2}\right] \Longrightarrow \theta_1 = \frac{\pi}{3} + \pi = \frac{4\pi}{3}.$$

So, $z_1 = 2\left[\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right]$. ■

Complex Numbers – Polar Form 2



2. $z_2 = 2 - i$

$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ with

$$r_2 = |z_2| = (z_2\bar{z_2})^{1/2} = (x_2^2 + y_2^2)^{1/2},$$

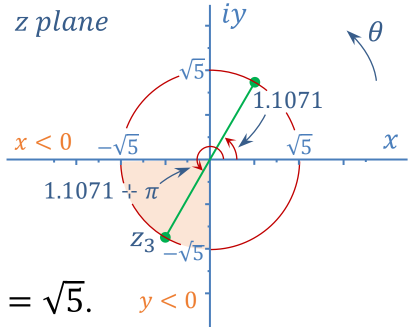
$$r_2 = [(2)^2 + (-1)^2]^{1/2} = [4 + 1]^{1/2} = \sqrt{5}.$$

$$\tan\theta_2 = \frac{y_2}{x_2} = \frac{-1}{2} = -\frac{1}{2} < 0 \Longrightarrow \begin{cases}\theta_2 \in \left[\frac{\pi}{2}, \pi\right] \\ \theta_2 \in \left[\frac{3\pi}{2}, 2\pi\right]\end{cases} \Longrightarrow \begin{cases}\theta_2 = \pi - 0.4636 \\ \theta_2 = 2\pi - 0.4636\end{cases}.$$

$$\begin{cases}y_2 < 0 \\ x_2 > 0\end{cases} \Longrightarrow \theta_2 \in \left[\pi, \frac{3\pi}{2}\right] \Longrightarrow \theta_2 = 2\pi - 0.4636 = 5.8196.$$

$$z_2 = \sqrt{5}[\cos(5.8196) + i\sin(5.8196)]. \blacksquare$$

Complex Numbers – Polar Form 3



3. $z_3 = -1 - 2i$

$z_3 = r_3(\cos\theta_3 + i\sin\theta_3)$ with

$$r_3 = |z_3| = (z_3\bar{z_3})^{1/2} = (x_3^2 + y_3^2)^{1/2},$$

$$r_3 = [(-1)^2 + (-2)^2]^{1/2} = [1 + 4]^{1/2} = \sqrt{5}.$$

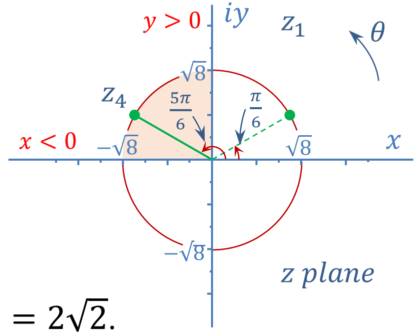
$$\tan\theta_3 = \frac{y_3}{x_3} = \frac{-2}{-1} = 2 > 0 \Longrightarrow \begin{cases} \theta_3 \in \left[0, \frac{\pi}{2}\right] \\ \theta_3 \in \left[\pi, \frac{3\pi}{2}\right] \end{cases} \Longrightarrow \begin{cases} \theta_3 = 1.1071 \\ \theta_3 = 1.1071 + \pi \end{cases}.$$

$$\begin{cases} y_3 < 0 \\ x_3 < 0 \end{cases} \Longrightarrow \theta_1 \in \left[\pi, \frac{3\pi}{2}\right] \Longrightarrow \theta_3 = 1.1071 + \pi = 4.2487.$$

So,

$$z_3 = \sqrt{5}[\cos 4.2487 + i\sin 4.2487]. \blacksquare$$

Complex Numbers – Polar Form 4



4. $z_4 = -\sqrt{6} + \sqrt{2}i$

$z_4 = r_4(\cos\theta_4 + i\sin\theta_4)$ with

$$r_4 = |z_4| = (z_4\bar{z}_4)^{1/2} = (x_4^2 + y_4^2)^{1/2},$$

$$r_4 = \left[\left(-\sqrt{6}\right)^2 + \left(\sqrt{2}\right)^2\right]^{1/2} = [6+2]^{1/2} = 2\sqrt{2}.$$

$$\tan\theta_4 = \frac{y_4}{x_4} = \frac{\sqrt{2}}{-\sqrt{6}} = -\frac{1}{\sqrt{3}} < 0 \Longrightarrow \begin{cases}\theta_4 \in \left[\frac{\pi}{2}, \pi\right] \\ \theta_4 \in \left[\frac{3\pi}{2}, 2\pi\right]\end{cases} \Longrightarrow \begin{cases}\theta_4 = \pi - \frac{\pi}{6} \\ \theta_4 = -\frac{\pi}{6}\end{cases}.$$

$$\begin{cases}y_4 > 0 \\ x_4 < 0\end{cases} \Longrightarrow \theta_4 \in \left[\pi, \frac{3\pi}{2}\right] \Longrightarrow \theta_4 = \pi - \frac{\pi}{6} = \frac{5\pi}{6}.$$

$$z_4 = 2\sqrt{2}\left[\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right]. \blacksquare$$