

## Polar Coordinates

Rectangular Coordinates  $(x, y) \rightarrow$  Polar Coordinates  $(r, \theta)$

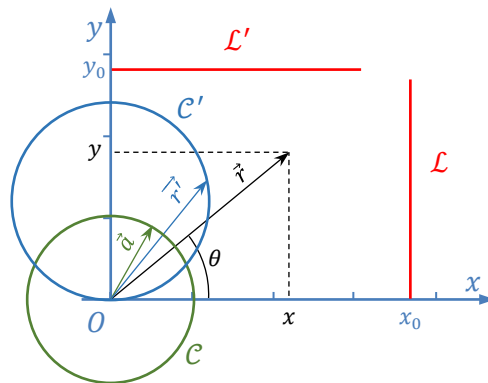
$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases} \rightarrow \begin{cases} r = \sqrt{x^2 + y^2} \\ \theta = \arctan \frac{y}{x} \end{cases}$$

$$L' \rightarrow r = \frac{y_0}{\sin \theta};$$

$$L \rightarrow r = \frac{x_0}{\cos \theta};$$

$$C' \rightarrow r' = a \cos \theta;$$

$$C \rightarrow a = cst.$$





## Example 1

### Double Integrals in Polar Coordinates

Integrate

$$f(x, y) = e^{-(x^2+y^2)}$$

over

$$\mathcal{D} = \{(r, \theta) \in \mathbb{R}^2: 0 \leq \theta \leq \pi, 0 \leq r \leq 2\}$$

$$\text{Answer: } \frac{\pi}{2} \left(1 - \frac{1}{e^4}\right)$$

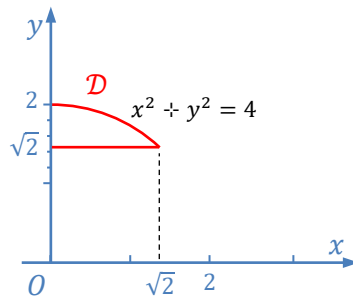
## Example 2

### Double Integrals in Polar Coordinates

Find the limits of integration for the region sketched below and rewrite the following integral in polar coordinates.

$$\iint_{\mathcal{D}} f(x, y) dx dy$$

$$\int_{?}^{?} \int_{?}^{?} ? dr d\theta$$





### Example 3

Double Integrals in Polar Coordinates

Find the area of the region that lies between the circle  $r = 1$  and the cardioid  $r = 1 + \cos \theta$  using polar coordinates.

$$\iint_{\mathcal{D}} f(x, y) dx dy$$

$$\int_{?}^{?} \int_{?}^{?} ? dr d\theta$$



## Example 4

### Double Integrals in Cylindrical & Spherical Coordinates

#### Double Integrals in Polar Coordinates

Find the area enclosed by the lemniscate  $r^2 = 4 \cos 2\theta$  using polar coordinates.

$$\int_{?}^{?} \int_{?}^{?} ? \, dr \, d\theta$$



## Multiple Integrals

### Triple Integral in $xyz$ Coordinates

### Exercise 1

Evaluate the volume in the first octant enclosed between the coordinate planes and the planes  $x + z = 1$  and  $x + z = 1$  using  $xyz$  Coordinates.