

### Series & Differential Equations L2 – Physics

1st Semester 2023/2024

# Example 1 Double Integrals

Evaluate the following iterated integrals.

1.

$$\int_{2}^{4} \int_{1}^{3} 40 - 2xy \, dx \, dy$$

2.

$$\int_{1}^{3} \int_{2}^{4} 40 - 2xy \, dy \, dx$$

Answer: 112

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# Example 2 Double Integrals

Find the volume of a solid bounded above by the plane z = 4 - x - y and below by the rectangle  $[0,1] \times [0,2]$ .

1.

$$\int_{0}^{2} \int_{0}^{1} 4 - x - y \, dx \, dy$$

2.

$$\int_0^1 \int_0^2 4 - x - y \, dy \, dx$$

Answer: 5 cubic units.

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# Example 3 Double Integrals

Suppose that the temperature (in degrees Celsius) at a point (x, y) on a flat metal plate is  $T(x, y) = 10 \div x^2 \div 3y^2$ , where x and y are in meters.

1. Find the average temperature  $T_{ave}$  of the rectangular portion of the plate for which  $0 \le x \le 1$  and  $0 \le y \le 2$ .

Answer:

$$T_s = \int_0^1 \int_0^2 10 \div x^2 \div 3y^2 \, dy \, dx = \frac{86}{3} \, ^{\circ}\text{C},$$
  
 $D = 1 \cdot 2 = 2 \, m^2, T_{ave} = \frac{T}{2} = \frac{43}{3} \, ^{\circ}\text{C}.$ 

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# Example 4 Double Integrals

Evaluate the following iterated integrals.

$$I = \int_0^{\ln 2} \int_{-1}^1 \sqrt{e^y + 1} \tan x \, dx \, dy$$

$$I = \int_0^{\ln 2} \int_{-1}^1 \sqrt{e^y + 1} \tan x \, dx \, dy = \int_{-1}^1 \tan x \, dx \int_0^{\ln 2} \sqrt{e^y + 1} \, dy.$$

$$\int_{-1}^1 \tan x \, dx = \int_{-1}^1 \frac{\sin x}{\cos x} \, dx = \int_{-1}^1 \frac{-1}{\cos x} \, d(\cos x).$$

$$\int_0^{\ln 2} \sqrt{e^y + 1} \, dy = \int_{\sqrt{2}}^{\sqrt{3}} \frac{2u^2}{u^2 - 1} \, du, \text{ with } u = \sqrt{e^y + 1} \Longrightarrow$$

$$y = \ln(u^2 - 1) \text{ and } dy = \frac{2u}{u^2 - 1} \, du \text{ hence } \int_0^{\ln 2} \to \int_{\sqrt{2}}^{\sqrt{3}} .$$

Answer: 0

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# Multiple Integrals Double Integrals

#### **Exercise 1**

- 1. Sketch a diagram of the region  $\mathcal{D}$  bounded by the curves  $y_1 = \sqrt{x}$  and  $y_2 = \sqrt{2(x-1)}$ .
- 2. Evaluate the following iterated integral.

$$\iint_{\mathcal{D}} x - y \, dy \, dx$$

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#### **Solution**

According to the graph below, we write:

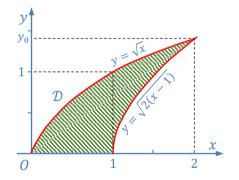
$$\int_0^1 \int_0^{\sqrt{x}} x - y \, dy \, dx + \int_1^2 \int_{\sqrt{2(x-1)}}^{\sqrt{x}} x - y \, dy \, dx$$

The intersection between the two curves occurs when  $y_1 = y_2$  at  $x_0 = 2$ ,  $y_0 = \sqrt{2}$ .

By changing the order of integration, we get:

$$\int_0^{\sqrt{2}} \int_{y^2}^{(y^2/2)+1} x - y \, dx \, dy$$

with  $x_1 = y^2$  and  $x_2 = \frac{y^2}{2} \div 1$ .



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### Series & Differential Equations L2 – Physics

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# Multiple Integrals Double Integrals

#### Exercise 2

1. Sketch the region over which the following integration takes place.

$$\int_{1}^{4} \int_{-x+3}^{2x} 2x - 1 \, dy \, dx$$

- 2. Evaluate the iterated integral.
- 3. Write the equivalent integral by reversing the order of integration.
- 4. Evaluate the resulting integral.

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$$\int_{1}^{4} \int_{-x+3}^{2x} 2x - 1 \, dy \, dx = \int_{1}^{4} (2x - 1)y \Big|_{-x+3}^{2x} \, dx,$$

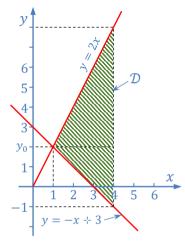
$$= \int_{1}^{4} \{ [2x - 1]2x - [2x - 1][-x + 3] \} \, dx,$$

$$= \int_{1}^{4} \{ 6x^{2} - 9x + 3 \} \, dx = \left( 2x^{3} - \frac{9x^{2}}{2} + 3x \right) \Big|_{1}^{4}.$$

$$= \frac{135}{2}.$$

$$\int_{-1}^{2} \int_{-y+3}^{4} 2x - 1 \, dx \, dy + \int_{2}^{8} \int_{y/2}^{4} 2x - 1 \, dx \, dy,$$

$$= \int_{-1}^{2} (x^{2} - x) \Big|_{-y+3}^{4} \, dy + \int_{2}^{8} (x^{2} - x) \Big|_{y/2}^{4} \, dy,$$



$$= \int_{-1}^{2} \{12 - (y^2 - 6y + 9) - y + 3\} dy + \int_{2}^{8} \left\{12 - \frac{y^2}{4} + \frac{y}{2}\right\} dy.$$

$$= \left(6y - \frac{y^3}{3} + \frac{5y^2}{2}\right)\Big|_{2}^{2} + \left(12y - \frac{y^3}{12} + \frac{y^2}{4}\right)\Big|_{2}^{8} = \frac{135}{2}.$$

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# Multiple Integrals Triple Integrals

#### Exercise 3

Use a triple integral to find the volume of the tetrahedron bounded by the four planes x = 0, z = 0, x = 2y, and  $x \div 2y \div z = 2$ .

$$\int_{?}^{?} \int_{?}^{?} \int_{?}^{?} f(x, y, z) dz dy dx$$
$$f(x, y, z) =?$$

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### Series & Differential Equations L2 – Physics

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# Multiple Integrals

# Triple Integrals

### Exercise 3

#### sketch

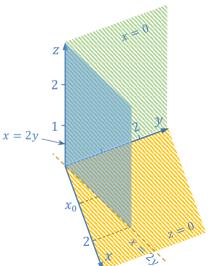
Four planes

$$x = 0$$
,

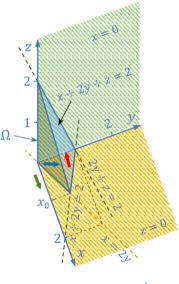
$$z = 0$$
,

$$x = 2y$$
,

$$x \div 2y \div z = 2$$
.







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$$\int_{?}^{?} \int_{?}^{?} \int_{?}^{?} f(x, y, z) \, dz \, dy \, dx$$

$$f(x, y, z) = 1$$

Starting with the inner integral, we see that z goes from 0 up to the bottom plane  $x \div 2y \div z = 2 \Longrightarrow$ 

$$\int_{0}^{2-x+2y} dz$$

Then, y goes from the curve  $y = \frac{1}{2}x$  up to the curve  $y = 1 - \frac{1}{2}x$ .

$$\rightarrow \int_{x/2}^{1-x/2} 2 - x \div 2y \, dy$$

x goes from 0 to  $x_0 = 1$ , the intersection between x + 2y = 2 and x = 2y.

$$\int_0^1 3 - 4x \div x^2 \, dx = \left( 3x - 2x^2 \div \frac{1}{3}x^3 \right) \Big|_0^1 = \frac{4}{3}$$

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