University of Boumerdes
Faculty of science
Physics' Department

Series & Differential Equations
L2 – Physics
1st Semester 2023/2024

Home Work 2

Multiple Integrals

Exercise 1

Simple & Double Integrals

Find the surface of the region limited by the curves $y = 2\sqrt{x}$ and $y = 4x - 2$ above the x axis.

1. Using simple integrals
2. Using double integrals with two orders of integration:
   a. y integration in the first order.
   b. x integration in the first order.

Home Work 2

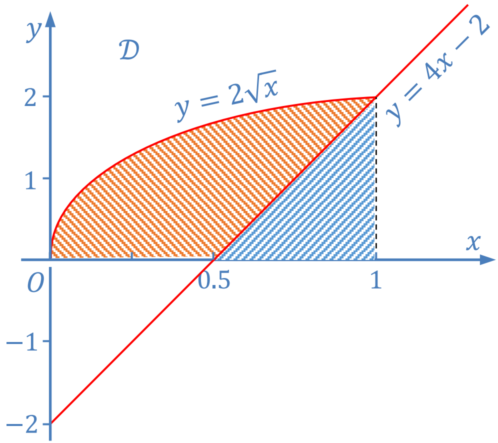
Solution of Exercise 1

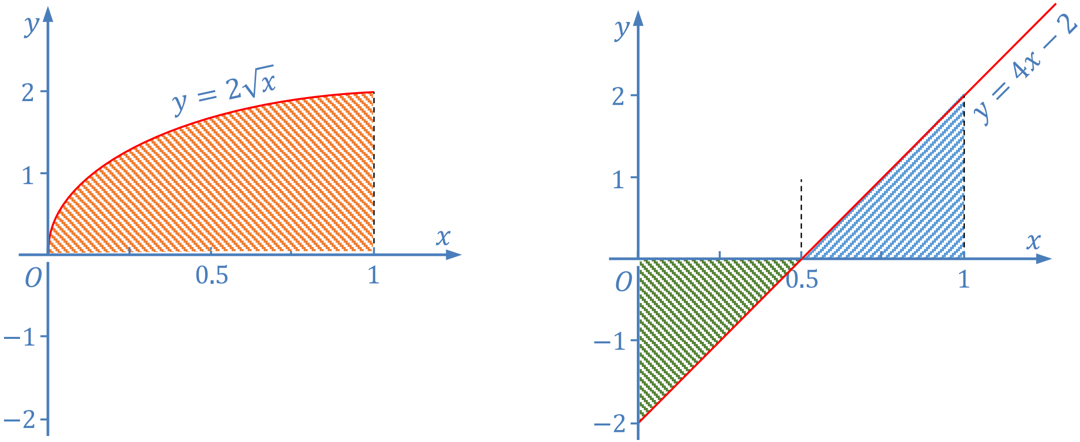
1. Simple Integrals

The region above the x axis is limited by $y_1(x) = 4x - 2$ and $y_2(x) = 2\sqrt{x}$.

The two surfaces merge, $y_2(x)$ contribute within the whole interval $[0,1]$, and $y_1(x)$ contribute only in $[0.5,1]$. We subtract the second from the first one.

$$S = \int_0^1 y_2(x)\,dx - \int_{0.5}^1 y_1(x)\,dx$$

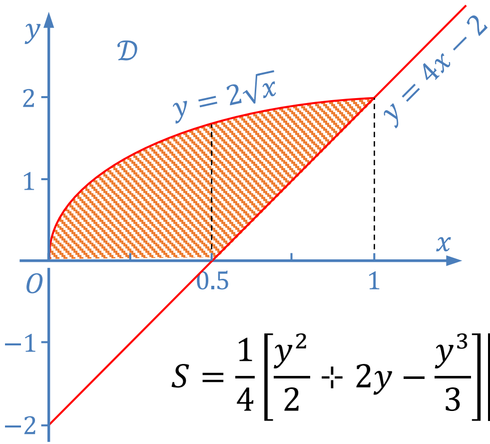




$$S = \int_0^1 2\sqrt{x}\,dx - \int_{0.5}^1 4x - 2\,dx = 2\frac{2}{3}x^{3/2}\Big|_0^1 - (2x^2 - 2x)|_{0.5}^1 = \frac{5}{6}$$

Home Work 2

Solution of Exercise 1



2a. Double Integrals $y \to x$

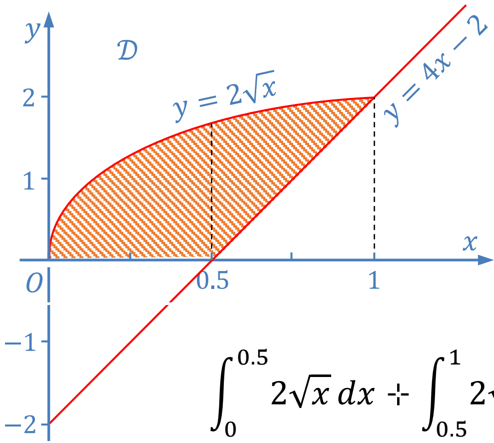
$$\int_0^2 \int_{y^2/4}^{(y+2)/4} dx\, dy = \int_0^2 x\Big|_{y^2/4}^{(y+2)/4} dx$$

$$S = \frac{1}{4}\int_0^2 (y+2) - y^2\, dy$$

$$S = \frac{1}{4}\left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]\Bigg|_0^2 = \frac{1}{4}\left[\frac{4}{2} + 2\cdot 2 - \frac{2^3}{3}\right] = \frac{5}{6}$$

Home Work 2

Solution of Exercise 1



$2b$. Double Integrals $x \to y$

$$\int_0^{0.5}\int_0^{2\sqrt{x}} dy\,dx + \int_{0.5}^{1}\int_{4x-2}^{2\sqrt{x}} dy\,dx$$

$$\int_0^{0.5} y\Big|_0^{2\sqrt{x}}\,dx + \int_{0.5}^{1} y\Big|_{4x-2}^{2\sqrt{x}}\,dx$$

$$\int_0^{0.5} 2\sqrt{x}\,dx + \int_{0.5}^{1} 2\sqrt{x}\,dx - \int_{0.5}^{1} (4x-2)\,dx$$

Home Work 2

Multiple Integrals

Exercise 2

Double Integrals

Evaluate the volume and sketch a simple plot of the region bounded above by the elliptical paraboloid surface $z = 10 + x^2 + 3y^2$, and below by the rectangle $[0,1] \times [0,2]$.

Home Work 2

Solution of Exercise 2

y then x

$$\int_0^1 \int_0^2 10 + x^2 + 3y^2 \, dy \, dx = \int_0^1 \int_0^2 10y + x^2 y + y^3 \big|_0^2 \, dx$$

$$\int_0^1 10 \cdot 2 + x^2 \cdot 2 + 2^3 \, dx = \int_0^1 28 + 2x^2 \, dx$$

$$28x + \frac{2}{3} x^3 \bigg|_0^1 = \frac{86}{3}$$

Home Work 2

Solution of Exercise 2

x then y

$$\int_0^2 \int_0^1 10 + x^2 + 3y^2 \, dx \, dy = \int_0^2 10x + \frac{1}{3}x^3 + 3xy^2 \Big|_0^1 dy$$

$$\int_0^2 10 + \frac{1}{3} + 3y^2 \, dy = \int_0^2 \frac{31}{3} + 3y^2 \, dy$$

$$\frac{31}{3}y + y^3 \Big|_0^2 = \frac{62}{3} + 8 = \frac{86}{3}$$

Home Work 2

Multiple Integrals

Exercise 3

Triple Integrals

Evaluate the volume and sketch a simple plot of the region bounded above by the elliptical paraboloid surface $z = 10 + x^2 + 3y^2$, and below by the rectangle $[0,1] \times [0,2]$.

Home Work 2

Solution of Exercise 3

$z = 10 + x^2 + 3y^2$, rectangle $[0,1] \times [0,2]$

$$0 \le z \le 10 + x^2 + 3y^2, 0 \le y \le 2, 0 \le x \le 1$$

$$\int_0^1 \int_0^2 \int_0^{10+x^2+3y^2} dz\, dy\, dx = \int_0^1 \int_0^2 z\Big|_0^{10+x^2+3y^2} dy\, dx$$

$$= \int_0^1 \int_0^2 10 + x^2 + 3y^2\, dy\, dx = \frac{86}{3}$$