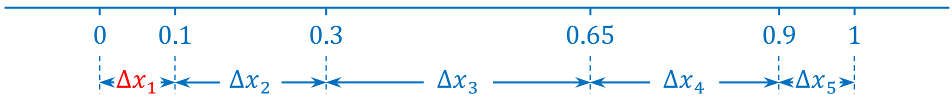
The Integral Sum 1

The integral sum of $f(x)$ within the partition $p = \{0, 0.1, 0.3, 0.65, 0.9, 1\}$,



is given by:

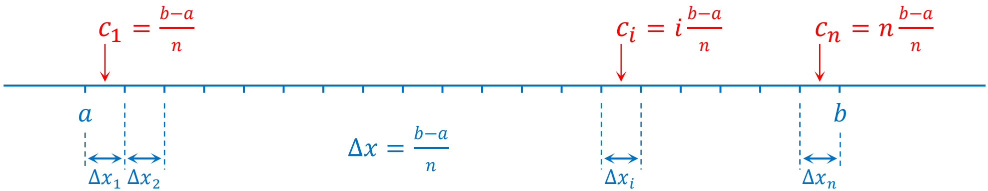
$$S = \sum_{i=1}^{n} f(c_i)\,\Delta x_i$$

$$S = f(c_1)\,0.1 + f(c_2)\,0.2 + f(c_3)\,0.35 + f(c_4)\,0.25 + f(c_5)\,0.1,$$

with $c_i \in I_i = [x_{i-1}, x_i]$ and $\Delta x_i = x_i - x_{i-1}$.

The Integral Sum 2

The integral sum of $f(x)$ within a partition $p = \{x_0, x_1, \dots, x_i, \dots, x_n\}$, is



$$S = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x_i$$

Uniform

with $\Delta x_i = \Delta x = (b-a)/n$ and $c_i = i\,(b-a)/n$.

The Mean Value

The mean value of $f(x)$ is approximately given by

$$\mu \sim \frac{f(c_1)+f(c_2)+f(c_3)+\cdots+f(c_i)+\cdots+f(c_n)}{n} \sim \sum_{i=1}^{n} f(c_i)\frac{1}{n} \sim \sum_{i=1}^{n} f(c_i)\frac{\Delta x_i}{b-a},$$

since $\Delta x = \frac{b-a}{n}$. If the partition Δx is smaller enough so that $n \to \infty$, thus

$$\mu = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\frac{\Delta x_i}{b-a} = \frac{1}{b-a} \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x_i$$

with $c_i = i\,(b-a)/n$. $f(x)$ is assumed to be continuous $\Longrightarrow \exists c \in [a,b]/$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x_i = (b-a)\mu = (b-a)\,f(c)$$

Calculus Theorem 1

Given a continuous function $f(t)$ within $[a, x]/\ \Delta t_i = \frac{x-a}{n}$. So, its integral sum

$$S = S_{a\to x} = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta t_i = F(x)$$

What is the meaning of $F(x)$? We add an amount Δx to the interval, thus:

$$S' = S_{a\to x+\Delta x} = \lim_{n'\to\infty} \sum_{i=1}^{n'} f(c_i)\,\Delta t_i = F(x + \Delta x)$$

Then we evaluate $F(x + \Delta x) - F(x) =?$

Calculus Theorem 2

$$F(x+\Delta x)-F(x)=\lim_{n'\to\infty}\sum_{i=1}^{n'} f(c_i)\,\Delta t_i\Big|_a^{x+\Delta x}-\lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\,\Delta t_i\Big|_a^{x}$$

$$F(x+\Delta x)-F(x)=\lim_{l\to\infty}\sum_{j=1}^{l} f(c_i)\,\Delta t_i\Big|_x^{x+\Delta x}=[(x+\Delta x)-x]\,f(c)$$

$$F(x+\Delta x)-F(x)=\Delta x\,f(c)\Longrightarrow f(c)=\frac{1}{\Delta x}\{F(x+\Delta x)-F(x)\}$$

$$\lim_{\Delta x\to 0} f(c)=\lim_{c\to x} f(c)=f(x)=\lim_{\Delta x\to 0}\frac{F(x+\Delta x)-F(x)}{\Delta x}=F'(x)$$

Calculus Theorem 3

Newton-Leibnitz notation

$$S_{a\to x} = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta t_i \Bigg|_a^x = \int_a^x f(t)\,dt = F(x) \dotplus C$$

If $x = a$ we get $S_{a\to a} = 0 = F(a) \dotplus C \Longrightarrow C = -F(a)$

Otherwise, if $x = b$ we get $S_{a\to b} = F(b) \dotplus C = F(b) - F(a)$. So

$$S_{a\to b} = \int_a^b f(x)\,dx = F(b) - F(a)\,, with\;\; F'(x) = f(x)$$